Course Code: 20HS0845 **R20**

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR
(AUTONOMOUS)
Siddharth Nagar, Narayanavanam Road – 517583

QUESTION BANK (DESCRIPTIVE)

Subject with Code: MATHEMATICAL AND STATISTICAL METHODS (20HS0845)
Course & Branch : II-B.Tech - Common to CSM & CIC

UNIT –I

Greatest Common Divisors and Prime Factorization

1	a	Using the principle of mathematical induction, to prove $1^3+2^3+3^3+.......+n^3=\left(\frac{n(n+1)}{2}\right)^2$.	[L1][CO1]	**[06M]**
	b	For every positive integer n, prove that 7^n-3^n is divisible by 4.	[L3][CO1]	**[06M]**
2	a	Find the Fibonacci number f_{13}.	[L1][CO1]	**[04M]**
	b	What is the sum of the first 11 terms of the give sequence 1, 1, 2, 3, 5, 8….	[L2][CO1]	**[06M]**
	c	Find the quotient and remainder in the division algorithm, with divisor 17 and dividend (i) 100 and (ii) -44	[L2][CO1]	**[06M]**
3	a	To multiply $(11101)_2$ and $(110001)_2$	[L2][CO1]	**[03M]**
	b	Find the gcd of 120 and 500.	[L2][[CO1]	**[03M]**
	c	Add $(ABAB)_{16}$ and $(BABA)_{16}$.	[L2][CO1]	**[03M]**
	d	Subtract $(434421)_5$ from $(4434201)_5$.	[L2][CO1]	**[03M]**
4	a	State and prove division algorithm. .	[L4][CO1]	**[08M]**
	b	Write the Euclidean algorithm.	[L3][CO1]	**[04M]**
5	a	If n is a composite integer, then n has a prime factor not exceeding √n.	[L2][CO1]	**[06M]**
	b	Using the formula for $\pi(n)$ find the number of primes ≤100.	[L2][CO1]	**[06M]**
6	a	Find the seven consecutive composite numbers.	[L3][CO1]	**[06M]**
	b	Find the gcd (414, 662) using Euclidean algorithm	[L4][CO1]	**[06M]**
7		State and prove fundamental theorem of arithmetic	[L5][CO1]	**[12M]**
8	a	For any positive integer n, prove that 8n+3 and 5n+2 are relatively prime	[L2][CO1]	**[06M]**
	b	Find the gcd and lcm of 504 and 540.	[L3][CO1]	**[06M]**
9	a	Find the general solution of $63x-23y=-7$. Using Euclidean algorithm	[L2][CO1]	**[06M]**
	b	Examine Whether the Linear Diophantine equation (LDE) $12x+13y=14$ is solvable. Write general solution if solvable	[L2][CO1]	**[06M]**
10	a	Prove that the Fermat number $F_5=2^{2^5}+1$ is divisible by 641.	[L4][CO1]	**[06M]**
	b	Factorize 809009 using Fermat's method of factorization.	[L3][CO1]	**[06M]**

UNIT-II

Congruences and Applications of congruences

1	a	Let a, b and m be integers such that $m > 0$ and (a, m) = d. If d does not divide b, the $ax \equiv b \pmod{m}$ has no solutions. If d divides b then $ax \equiv b \pmod{m}$ has exactly d Congruent solutions modulo m.	[L3][CO2]	**[06M]**
	b	To find all solutions of $9x \equiv 12 \pmod{15}$.	[L2][CO2]	**[06M]**
2	a	Solve the congruence $6x \equiv 3 \pmod{9}$.	[L2][CO2]	**[04M]**
	b	State and prove Chinese remainder theorem.	[L4][CO2]	**[08M]**
3	a	State and prove Wilson's theorem.	[L3][CO2]	**[04M]**
	b	Show that 18! +1 is divisible by 437.	[L4][CO2]	**[08M]**
4	a	State and prove Fermat's theorem	[L2][CO2]	**[04M]**
	b	Solve system of linear equations $3x + 4y \equiv 5 \pmod{13}$ $2x + 5y \equiv 7 \pmod{13}$.	[L4][CO2]	**[08M]**
5	a	State and prove Euler theorem.	[L3][CO2]	**[06M]**
	b	Compute the least residue of $2^{340} \pmod{341}$.	[L2][CO2]	**[06M]**
6		Solve the system of congruence $x \equiv 3 \pmod{10}$, $x \equiv 8 \pmod{15}$, $x \equiv 5 \pmod{84}$)	[L6][CO2]	**[12M]**
7	a	Find $\sigma(200)$ and $\tau(200)$, where $\sigma(n)$ denotes sum of the divisors and τ(n) denotes number of divisors.	[L3][CO2]	**[03M]**
	b	If $\phi(n)$ denotes the number of positive integers less than or equal to n, then find φ(28)?	[L1][CO2]	**[03M]**
	c	Find the remainder of 17! When divided by 23.	[L2][CO2]	**[06M]**
8	a	Find the remainder when $1! + 2! + 3! + 4! + ... + 300!$ is divided by 13.	[L2][CO2]	**[06M]**
	b	Let P be odd then $1^2.3^2.5^2..........(p-2)^2 \equiv (-1)^{\frac{p+1}{2}} \pmod{p}$	[L2][CO2]	**[06M]**
9	a	Show that 63! ≡-1(mod 71).	[L3][CO2]	**[06M]**
	b	Find the remainder when 15^{1976} is divided by 23.	[L2][CO2]	**[06M]**
10		Solve the polynomial congruence $2x^3 + 7x - 4 \equiv 0 \pmod{200}$	[L4][CO2]	**[12M]**

Course Code: 20HS0845 **R20**

UNIT-III

ESTIMATION

1	a	Prove that for a random sample of size n, $x_1, x_2, x_3 .... x_n$ taken from a finite population $S^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$ is not unbiased estimator of the parameter σ^2 but $\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$ is unbiased .	[L6][CO3]	**[06M]**
	b	If $x_1, x_2, x_3 ...... x_n$ is a random sample from a normal population N (μ,1). Show that $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of μ^2+1.	[L5][CO3]	**[06M]**
2	a	A sample of size 10 was taken from a population standard deviation of sample is 0.03. Find the maximum error with 99% confidence.	[L3][CO3]	**[06M]**
	b	If we can assert with 95% that the maximum error is 0.05 and p=0.2. Find the sample size.	[L4][CO3]	**[06M]**
3	a	The mean and the standard deviation of a population are 11795 and 14054 respectively. If n=50, find 95% confidence interval for the mean? And what is the maximum error we can assert at 95% confidence level?	[L2][CO3]	**[06M]**
	b	Find 95% confidence limits for the mean of a normality distributed population from which the following sample was taken 15, 17, 10, `18, 16, 9,7,11,1 ,14.	[L4][CO3]	**[06M]**
4	a	Drying times for paint 3.4, 2.5, 4.8, 2.9, 3.6, 2.8, 3.3, 5.6, 3.7, 2.8, 4.4, 4.0, 5.2, 3.0, 4.8. Find a 95% prediction interval for drying of the next trail of paint.	[L4][CO3]	**[06M]**
	b	In a random sampling from normal population $N(\mu, \sigma^2)$ find the maximum likelihood estimators for (i) μ and σ^2 is known (ii) σ^2 when μ is known and (iii) the simultaneous estimation of μ and σ^2.	[L2][CO3]	**[06M]**
5	a	Define the tolerance interval and explain exact tolerant interval and exact nonparametric tolerance interval.	[L2][CO3]	**[06M]**
	b	What is the size of the smallest sample required to estimate an unknown proportion to within a maximum error of 0.06 with at least 95% confidence?	[L2][CO3]	**[06M]**
6		The mean of a random sample is an unbiased estimate of the man of population 3, 6, 9, 15, 27. (a) List of all possible samples of size 3 that can be taken without replacement from the finite population? (b) Calculate the mean of each of the sample listed in (a) and assigning each sample a probability of 1/10. Verify that the man of these X is equal to the mean of the population θ. Prove that $\bar{x}$ is an unbiased estimate of θ	[L4][CO3]	**[12M]**
7	a	Show that the sample variance is a consistent estimator of the population variance σ^2.	[L3][CO3]	**[06M]**
	b	Show that $ns^2/n-1$ is a consistent estimator of σ^2.	[L5][CO3]	**[06M]**
8		What is mean by point estimation? Define the following estimators and give an example for each (i) Unbiased estimator (ii) Consistent estimator (iii) Efficient estimator (iv) Sufficient estimator.	[L5][CO3]	**[12M]**
9		Explain (i) confidence interval for the mean of the normal population (ii) confidence interval for the difference between the means (iii)confidence interval for the proportion (iv) Confidence interval for the difference between tow proportions.	[L6][CO3]	**[12M]**
10	a	Let $x_1, x_2, x_3 .... x_n$ denote random sample of size n from a uniform population with probability density function $f(x,\theta) = 1; \; \theta - 1/2 \le x \le \theta + 1/2, \; -\infty < \theta < \infty$. Obtain Maximum likely estimation for θ	[L3][CO3]	**[06M]**
	b	Obtain the maximum likelihood estimation of θ in $f(x,\theta) = (1+\theta)\, x^{\theta}, \; 0 < x < 1$ based on an independent sample of size n. Examine whether this estimate is sufficient for θ.	[L4][CO3]	**[06M]**

UNIT-IV

STOCHASTIC PROCESS AND MARKOV PROCESS

1	a	Suppose a communication system transmits the digits 0 and 1 through many stages. At each state the probability that the same digit will be received by the next stage as transmitted, is 0.75. What is the probability that a 0 is entered at the first stage is received as a 0 in the 5th stage?	[L3][CO4]	**[06M]**
	b	Let $P=\begin{pmatrix}\frac{3}{4} & \frac{1}{4}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}$ be the transition probability matrix of a two state Markov chain. Find the stationary probabilities of the chain.	[L2][CO4]	**[06M]**
2	a	The transition probability matrix of a Markov chain $\{x_n\}$, n=1, 2, 3,…having three states, 1,2 and 3is $P=\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$ and the initial distribution is $P^{(0)}=(0.1,\ 0.2,\ \ 0.1)$. Find $(i)\ P(X_2=3,\ X_1=3,\ X_0=2)\quad (ii)\ P(X_2=3)$ $(iii)\ P(X_2=2,\ X_2=3,\ X_1=3,\ X_0=2)$	[L5][CO4]	**[6M]**
	b	A college student X has the following study habits. If he studies one night, he is 70% sure nit to study the next night. If he does not study one night, he is only 60% sure not to study the next night also. Find (i) the transition probability matrix (ii) how often he studies in the long run.	[L2][CO4]	**[06M]**
3	a	Explain Classification of Markov chains.	[L4][CO3]	**[08M]**
	b	(b) Consider a Markov chain with the state space $\{0.1\}$ and transition probability matrix $P=\begin{pmatrix}0 & 1\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}$ (i) Is the state 0 recurrent? Explain (ii) Is the state 1 transient? Explain?	[L2][CO2]	**[04M]**
4	a	Three boys A, B, C are throwing a ball to each other. A always through the ball to B and B always throws to C but C is just as likely to throw the ball to B as to A. show that the process is Markovian. Find the transition matrix and classify the states.	[L4][CO4]	**[06M]**
	b	Find the nature of the states of the Markov chain with the transition probability matrix $P=\begin{bmatrix}0 & 1 & 0\\ \frac{1}{2} & 0 & \frac{1}{2}\\ 0 & 1 & 0\end{bmatrix}$	[L4][CO4]	**[06M]**
5		A fair dice is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses, find the transition probability matrix p of the Markov chain $\{X_n\}$. Find also $P\{X_2 = 6\}$ and P^2.	[L4][CO4]	**[12M]**
6		Classification of states of a Markov chain and give the example	[L2][CO4]	**[12M]**

7		A gambler has Rs.2. He bets Re.1 at a time and wins Re.1 with probability ½. He stops playing if he loses Rs.2 or wins Rs.4. (a) what is the transition probability matrix of the related Markov chain? (b) What is the probability that he has lost his money at the end of 5 plays? What is the probability that the game lasts more than 7 plays?	[L5][CO4]	**[12M]**
8	There are two boxes, box I contains 2 white balls and box II contains 3 red balls. A each step of the process, a ball is selected from each box and the 2 balls are Interchanged. Thus box 1 always contains 2 balls and box II always contains 3 balls. The states of the system represent the number of red balls in box I after the interchange. Find (i) the transition matrix of the system (ii) the probability that there are 2 red balls in the box I after 3 steps and (iii) the probability that, in the long run there are 2 red balls in box I.		[L4][CO4]	**[12M]**
9	a	State Chapman – Kolmogorov equation	[L2][CO4]	**[04M]**
	b	A man either drives a car or catches a train to go the office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if 6 appeared. Find (i) the probability that he takes a train on the third day and (ii) the probability that he drives to work in the long run.	[L5][CO4]	**[08M]**
10		Write a assumption and properties of Markov process and also give the example.	[L4][CO4]	**[12M]**

UNIT-V

QUEUEING THEORY

1	Explain Pure Birth and Death process. Also Derive the formulas for (M/M/1); (∞/FCFS) model.	[L5][CO5]	**[12M]**
2	Explain queuing theory and its characteristic	[L6][CO5]	**[12M]**
3	Self-service canteen employee's one cashier at its counter 8 customers arrives per every 10 minutes on an average. The cashier can serve on average one per minute. Assuming that the arrivals are Poisson and the service time distribution is exponential, determine (i) The average number of customers in the system. (ii) The average queue length (iii) Average time a customer spends in the system. (iv) Average waiting time of each customer.	[L5][CO5]	**[12M]**
4	A tollgate is operated on a freeway where cars arrive according to a Poisson distribution with mean frequency of 1.2 cars per minute. The time of completing payments follows an exponential distribution with payments follows an exponential distribution with mean of 20 seconds. Find (i) The idle time of the counter (ii) Average number of cars in the system (iii) Average number of cars in the queue (iv) Average time that a car spends in the system (v) Average time that a car spends in the queue. (vi) The probability that a car spends more than 30 seconds in the system.	[L6][CO5]	**[12M]**
5	A one person barber shop has six chairs to accommodate people waiting for haircut. Assume that customers who arrive when all the six chairs are full leave without entering the shop. Customers arrive at the average of 3 per hr and spend an average of 15minutes for service. Find (a) The probability that a customer can get directly into the barber chair upon arrival. (b) Expected number of customers waiting for a haircut. (c) Effective arrival rate. (d) The time a customer can expect to spend in the barber shop.	[L6][CO5]	**[12M]**
6	At a railway station only one train is handled at a time. The railway yard is sufficient only for two trains to wait while other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hr and the railway station can handled them on an average of 12 per hr. Assuming Poisson arrivals and exponential service distribution, find the steady state probabilities for the various number of trains in the system. Find also the average waiting time of a new train coming into the yard.	[L5][CO6]	**[12M]**
7	Satyam info way has two persons for its browsing Centre. If the service time for each client is exponential with mean 4 minutes, and if people arrive in a Poisson fashion at the rate of 10 an hour. Then calculate the (a) Probability of having to wait for service (b) Expected percentage of idle time for each girl (c) If a client has to wait, what is the expected length of his waiting time?	[L5][CO5]	**[12M]**
8	Four counters are being used in the frontier of a country to check the passports and necessary papers of the tourists. The tourists choose a counter at random. If the arrivals at the frontier are Poisson at the rate λ and the service time is exponential with parameter $\frac{\lambda}{2}$, what is the steady state average queue at each counter?	[L5][CO5]	**[12M]**
9	A car servicing station has two bays where service can be offered simultaneously. Due to space limitation only four cars are accepted for servicing. The arrival pattern is Poisson with 12 cars per day. The service time in both the bays is exponentially distributed with μ=8 cars per day per bay. Find the average number of cars in the service station the average number of cars waiting to be serviced and the average time spends in the system.	[L6][CO5]	**[12M]**

10	Arrival rate of telephone calls at a telephone booth are according to Poisson distribution with an average tome of 12 min between two consecutive call arrivals. The length of telephone calls is assumed to be exponential distributed with mean 4 minutes. Find (i) Find the average queue length that forms from time to time. (ii) Probability that a caller arriving at the booth will have to wait. (iii) What is the probability that an arrival will have to wait for more than 15 minutes before the phone is free. (iv)Find the fraction of a day that the phone will be in use.	[L6][CO5]	**[12M]**

Prepared by:

Dr. A. Vijayalakshmi
Associate Professor/BS&H